Les filtres

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Définition : C'est un quadripôle qui transmis les signaux (tensions) de fréquences comprises dans un certain domaine.

* passe bas passif
* passe bas actif.
* passe haut passif.
* passe bande.

* **transmittance :** $T = \frac{U_{sm}}{U_{em}}$.

* **Gain :** $G = 20 \log T$.

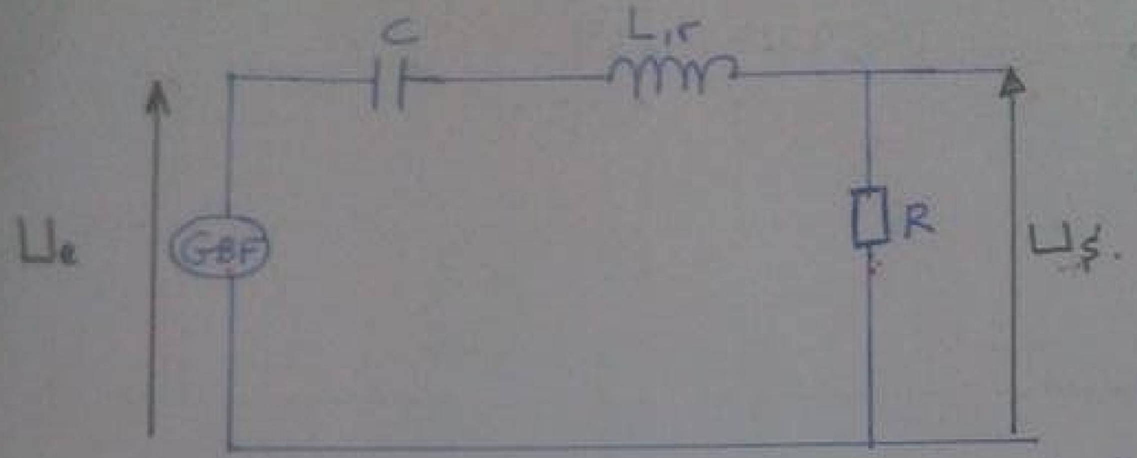
* **Bande passante :** $G \geq G_0 - 3\,dB$

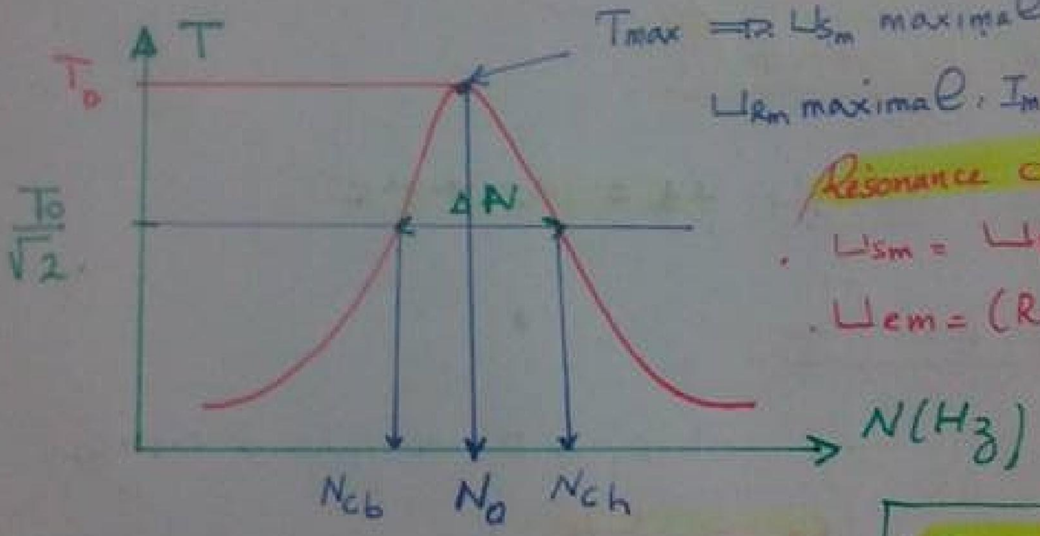
  $T \geq \frac{T_0}{\sqrt{2}}$.

* **fréquence de coupure :**

$$N = N_c \longrightarrow \begin{cases} G = G_0 - 3\,dB \\ T = \frac{T_0}{\sqrt{2}} \end{cases}$$

I - passe bande





$T_{max} \Rightarrow U_{s_m}$ maximale

U_{R_m} maximale, I_m maximale

Résonance d'intensité

. $U_{sm} = U_{Rm} = R\,I_m$

. $U_{em} = (R+r)\,I_m$.

$$T_0 = \frac{R}{R+r}$$

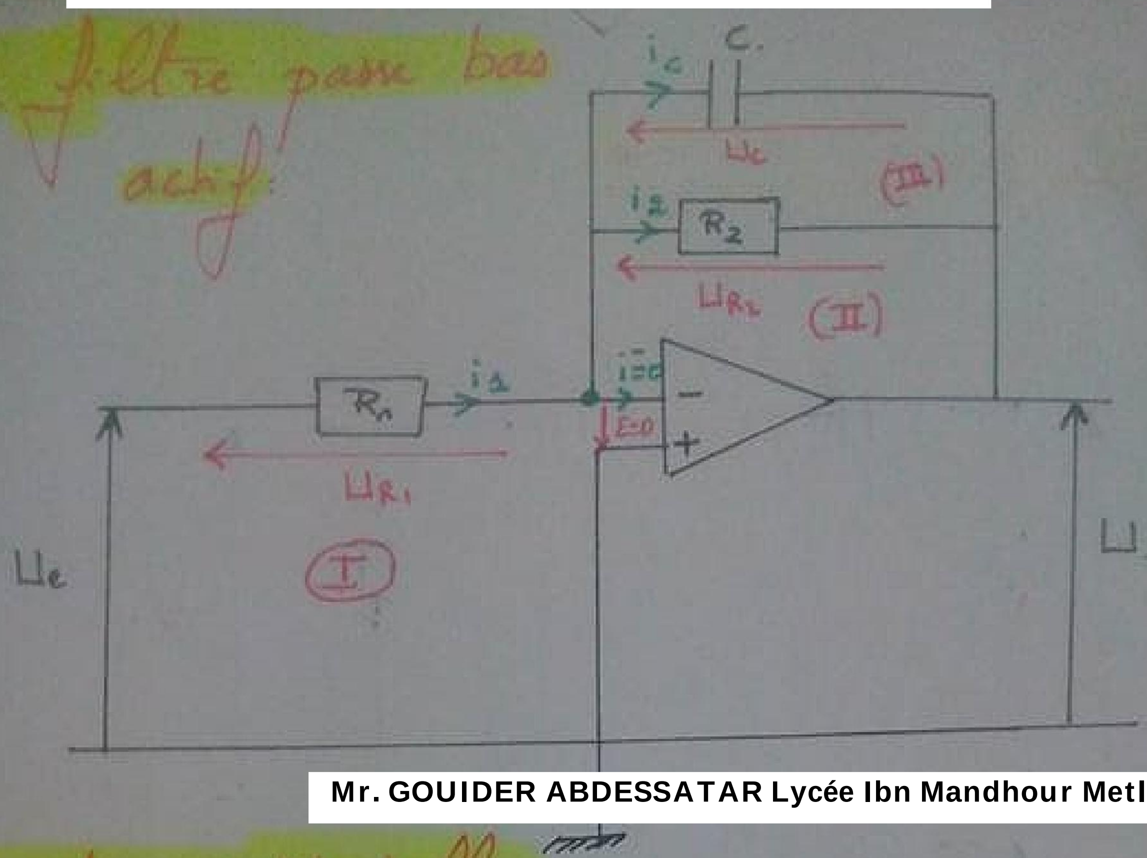
Bande passante : $[N_{cb} - N_{ch}]$

Largeur de la bande passante : $\Delta N = N_{ch} - N_{cb}$

facteur de surtension : $Q = \frac{N_0}{\Delta N}$.

On rappel que $Q = \frac{1}{R+r}\sqrt{\frac{L}{C}}$.

III- filtre passe bas actif :



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1) Équation différentielle :

. Loi des noeuds : $i_1 = i_2 + i_C$.

. Loi des mailles :

(I) : $u_e - u_{R_1} = 0 \Rightarrow u_e = R_1 i_1$

$$i_1 = \frac{u_e}{R_1}$$

(II) : $u_s + u_{R_2} = 0 \Rightarrow u_s = -u_{R_2}$

$u_s = -R_2 i_2 \rightarrow$ $i_2 = -\frac{u_s}{R_2}$

(III) $u_s + u_C = 0 \Rightarrow u_s = -u_C = -\frac{q}{C}$

$q = -C\,u_s$; $i = \frac{dq}{dt}$

$$i_C = -C\frac{du_s}{dt}$$

$$\frac{U_e}{R_1} = -\frac{U_s}{R_2} - C\frac{dU_s}{dt}.$$

$$-\frac{R_1}{R_2}U_s - R_A C\frac{dU_s}{dt} = U_e.$$

$$\boxed{\frac{R_1}{R_2}U_s + R_A C\frac{dU_s}{dt} = -U_e.}$$

2) Construction de Fresnel :

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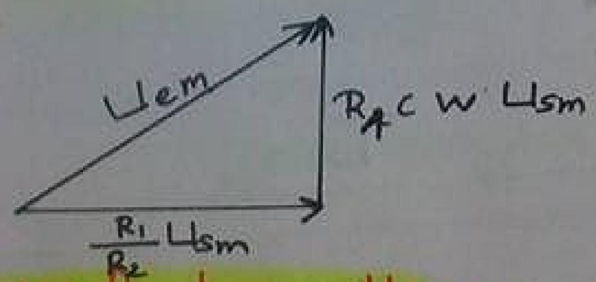
	$\frac{R_1}{R_2}U_s$	$R_A C\frac{dU_s}{dt}$	$-U_e$
Amplitude	$\frac{R_1}{R_2}U_{sm}$	$R_A C w U_{sm}$	U_{em}
Phase.	φ_s	$\varphi_s + \pi/2$	$\varphi_e + \pi$



3) Expression de la transmittance :

$$\left(\frac{R_1}{R_2}U_{sm}\right)^2 + (R_A C w U_{sm})^2 = U_{em}^2$$

$$U_{sm}^2\left[\left(\frac{R_1}{R_2}\right)^2 + (R_A C w)^2\right] = U_{em}^2.$$

$$\left(\frac{R_1}{R_2}\right)^2 + (R_A C w)^2 = \frac{U_{em}^2}{U_{sm}^2}$$

$$\frac{U_{sm}^2}{U_{em}^2} = \frac{1}{\left(\frac{R_1}{R_2}\right)^2 + (R_1 C w)^2}$$

$$\frac{U_{sm}^2}{U_{em}^2} = \frac{1}{\left(\frac{R_1}{R_2}\right)^2 \left[1 + (R_2 C w)^2\right]}$$

$$\frac{U_{sm}^2}{U_{em}^2} = \frac{\left(\frac{R_2}{R_1}\right)^2}{1 + (2\pi R_2 C N)^2}.$$

$$T = \frac{U_{sm}}{U_{em}} = \frac{\frac{R_2}{R_1}}{\sqrt{1 + (2\pi R_2 C N)^2}}.$$

Soit pour $N = 0$ $T_0 = \frac{R_2}{R_1}.$

$$T = \frac{T_0}{\sqrt{1 + (2\pi R_2 C N)^2}}.$$

4) Expression de Gain G : $G = 20 \log T$

$$\log \frac{a}{b} = \log a - \log b.$$

$$\log a^n = n \log a.$$

$$\log 1 = 0$$

$$\sqrt{x} = x^{1/2}$$

$$G = 20 \log \frac{T_0}{\left(1 + (2\pi R_2 C N)^2\right)^{1/2}}.$$

$$G = 20 \log T_0 - 20 \log \left(1 + (2\pi R_2 C N)^2\right)^{1/2}$$

$$G = 20 \log T_0 - 10 \log \left(1 + (2\pi R_2 C N)^2\right)$$

si $N = 0$ $\quad G_0 = G_{max} = 20 \log T_0$

$$G = G_0 - 10 \log \left(1 + (2\pi R_2 C N)^2\right)$$

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(1) Condition pour qu'un filtre soit passante

$$G \geq G_0 - 3\ dB.$$

$$G_0 - 10 \log\left(1 + (2\pi R_2 C N)^2\right) \geq G_0 - 3\ dB$$

$$\log\left(1 + (2\pi R_2 C N)^2\right) \leq 0{,}3.$$

$$1 + (2\pi R_2 C N)^2 \leq 10^{0{,}3} = 2.$$

$$(2\pi R_2 C N)^2 \leq 1.$$

$$2\pi R_2 C N \leq 1$$

$$\boxed{N \leq \frac{1}{2\pi R_2 C}}$$

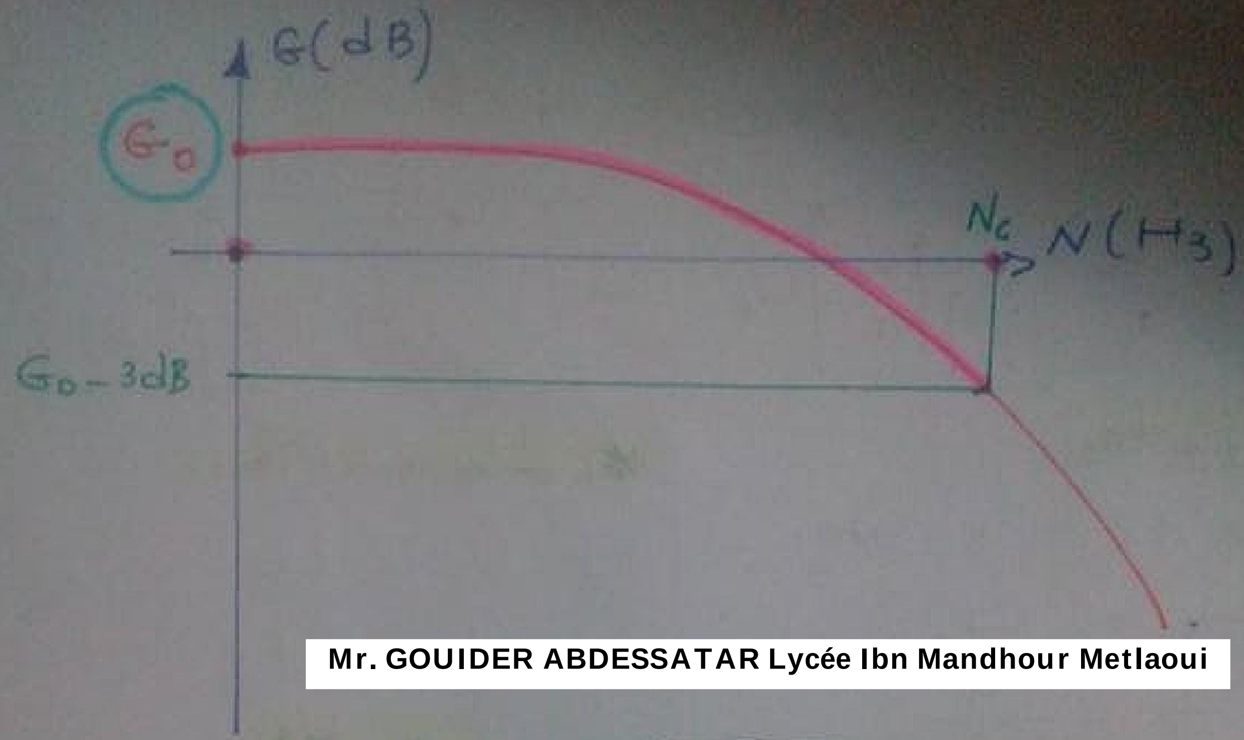
Bande passante : $\left[0 - \frac{1}{2\pi R_2 C}\right]$

et $N_c = \frac{1}{2\pi R_2 C}$.

$[0 - N_c]$

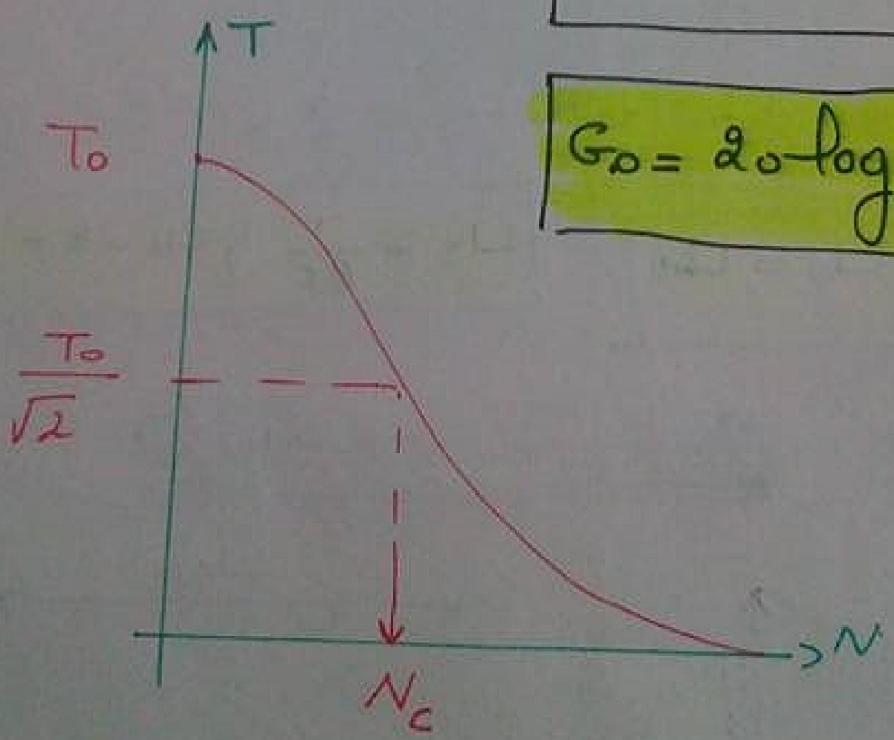
transmis les basses fréquences ⇒ filtre passe bas

(7)



$$N_c = \frac{1}{2\pi R_2 C}$$

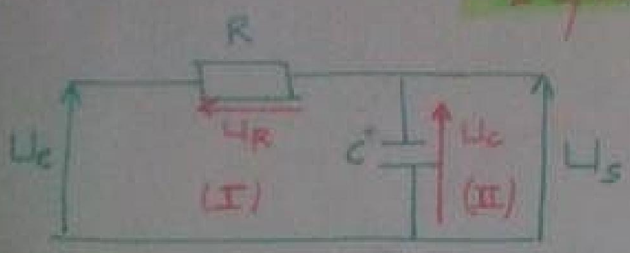
$$G_0 = 20 \log \frac{R_2}{R_1}$$

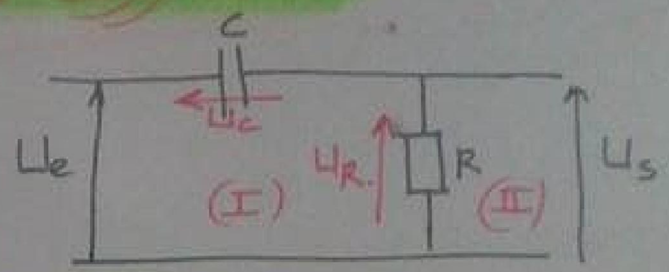


Filtre passe bas passif

Filtre passe haut passif

Equation différentielle





Filtre passe bas – Loi des mailles :

(I) $u_R + u_C = u_e$

(II) $u_s = u_C$

$u_s = \frac{q}{C}$

$q = C\,u_s$

$R\,i + u_s = u_e$

$R\frac{dq}{dt} + u_s = u_e$

$$RC\frac{du_s}{dt} + u_s = u_e$$

Filtre passe haut – Loi des mailles :

(II) : $u_s = u_R$.

$u_s = R\,i$.

$i = \frac{u_s}{R}$.

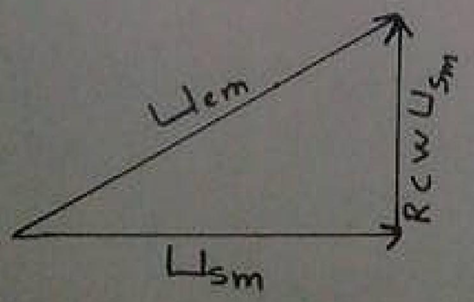
(I) . $u_R + u_C = u_e$.

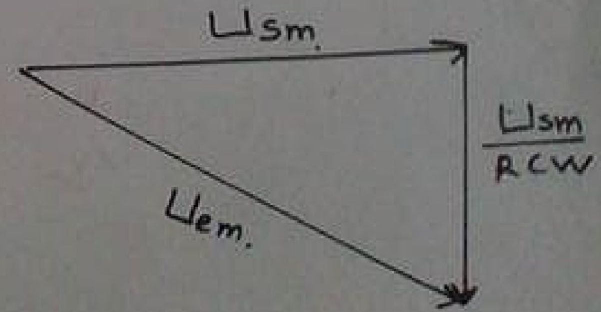
$u_s + \frac{q}{C} = u_e$.

$u_s + \frac{1}{C}\int i\,dt = u_e$.

$$u_s + \frac{1}{RC}\int u_s\,dt = u_e$$

Construction de Fresnel





Transmittance

$$U_{sm}^2 + (RC\omega\, U_{sm})^2 = U_{em}^2.$$

$$U_{sm}^2\left(1 + (RC\omega)^2\right) = U_{em}^2$$

$$\frac{U_{sm}^2}{U_{em}^2} = \frac{1}{1 + (RC\omega)^2}$$

$$T = \frac{1}{\sqrt{1 + (2\pi RCN)^2}}$$

$$U_{sm}^2 + \left(\frac{U_{sm}}{RC\omega}\right)^2 = U_{em}^2.$$

$$U_{sm}^2\left(1 + \frac{1}{(RC\omega)^2}\right) = U_{em}^2.$$

$$\frac{U_{sm}^2}{U_{em}^2} = \frac{1}{1 + \frac{1}{(RC\omega)^2}}.$$

$$T = \frac{1}{\sqrt{1 + \frac{1}{(2\pi RCN)^2}}}$$

Gain G

$$G = 20 \log T$$

$$G = 20 \log \frac{1}{\left(1 + (2\pi RCN)^2\right)^{1/2}}$$

$$= \underbrace{20 \log 1}_{=0} - 20 \log\left(1 + (2\pi RCN)^2\right)^{1/2}$$

$$G = -10 \log\left(1 + (2\pi RCN)^2\right)$$

$$G = 20 \log T.$$

$$G = 20 \log \frac{1}{\left(1 + \frac{1}{(2\pi RCN)^2}\right)^{1/2}}$$

$$= \underbrace{20 \log 1}_{=0} - 20 \log\left(1 + \frac{1}{(2\pi RCN)^2}\right)^{1/2}$$

$$G = -10 \log\left(1 + \frac{1}{(2\pi RCN)^2}\right)$$

Bande passante

$G \geq G_0 - 3\,dB$

$(G_0 = 0$

$-10\log\left(1+(2\pi RCN)^2\right) \geq -3$

$\log\left(1+(2\pi RCN)^2\right) \leq 0{,}3$

$1+(2\pi RCN)^2 \leq 2$

$2\pi RCN \leq 1$

$N \leq \dfrac{1}{2\pi RC}$

Bande passante :

$\left[0 - \dfrac{1}{2\pi RC}\right]$

$N_c = \dfrac{1}{2\pi RC}$

$[0 - N_c]$

+ Ce filtre transmis les signaux des basses fréquences

→ filtre passe bas passif

— formé par des dipôles passifs

$-10\log\left(1+\dfrac{1}{(2\pi RCN)^2}\right) \geq -3$

$\log\left(1+\dfrac{1}{(2\pi RCN)^2}\right) \leq 0{,}3$

$1+\dfrac{1}{(2\pi RCN)^2} \leq 2$

$\dfrac{1}{2\pi RCN} \leq 1$

$2\pi RCN \geq 1$

$N \geq \dfrac{1}{2\pi RC}$

Bande passante

$\left[\dfrac{1}{2\pi RC}, +\infty\right[$

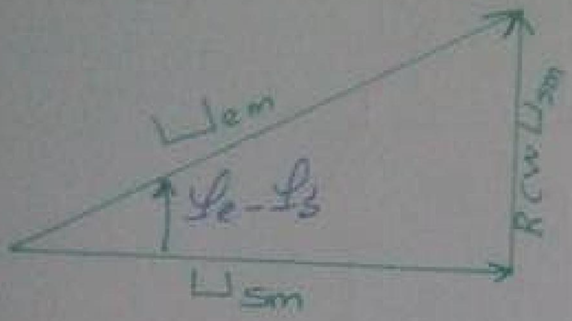
$N = \dfrac{1}{2\pi RC}$.

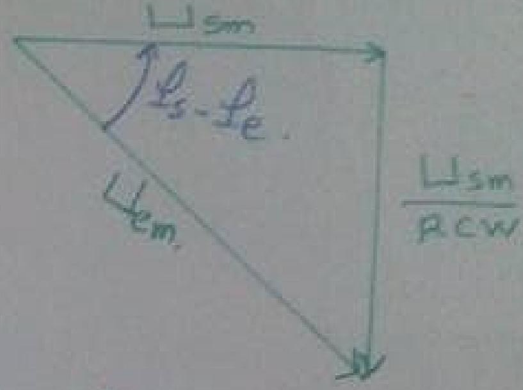
$[N_c, +\infty[$

Ce filtre transmis les hautes fréquences

→ filtre passe haut passif

Déphasage :





$tg(\varphi_e - \varphi_s) = \frac{RCW.U_{sm}}{U_{sm}}$

$tg(\varphi_e - \varphi_s) = RCW$

$= 2\pi RCN$

$tg(\varphi_e - \varphi_s) = \frac{N}{N_c}$.

Si $N = N_c$.

$tg(\varphi_e - \varphi_s) = 1$

$\varphi_e - \varphi_s = \frac{\pi}{4}$ rad.

$U_e(t)$ est en avance de phase % à $U_s(t)$

$tg(\varphi_s - \varphi_e) = \frac{\frac{U_{sm}}{RCW}}{U_{sm}}$

$= \frac{1}{RCW}$

$= \frac{1}{2\pi RCN}$

$tg(\varphi_s - \varphi_e) = \frac{N_c}{N}$.

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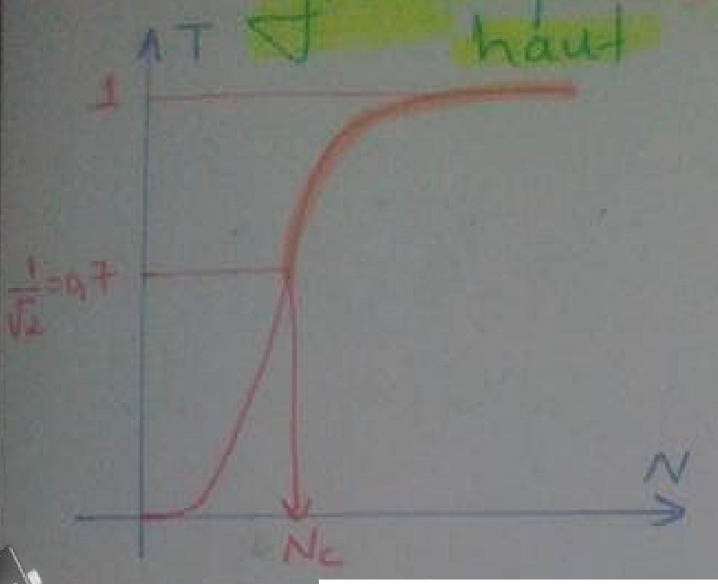
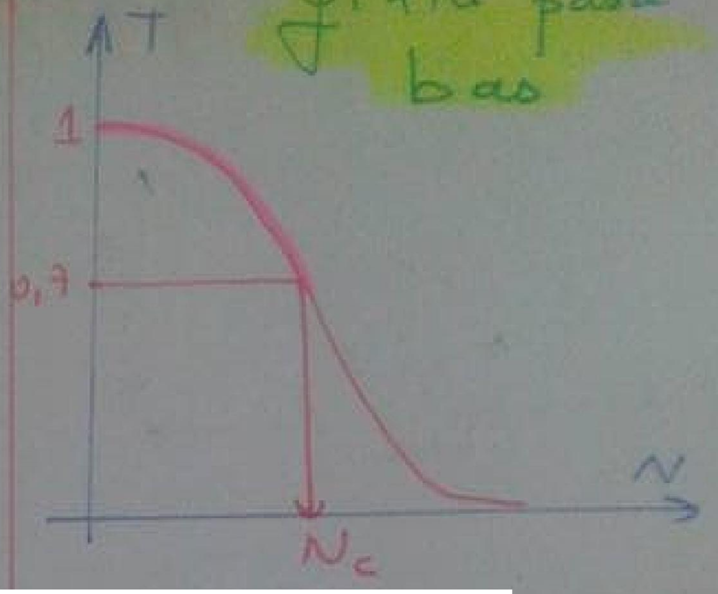
Si $N = N_c$

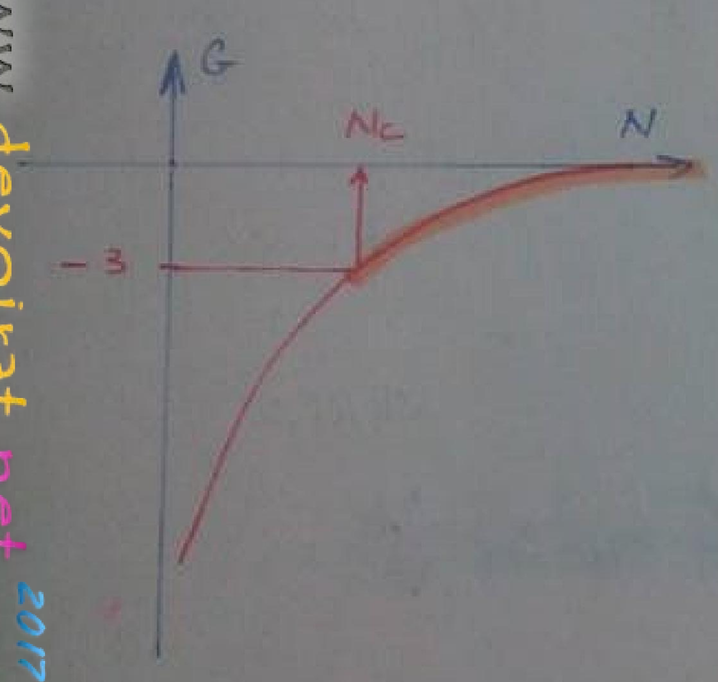
$tg(\varphi_s - \varphi_e) = 1$

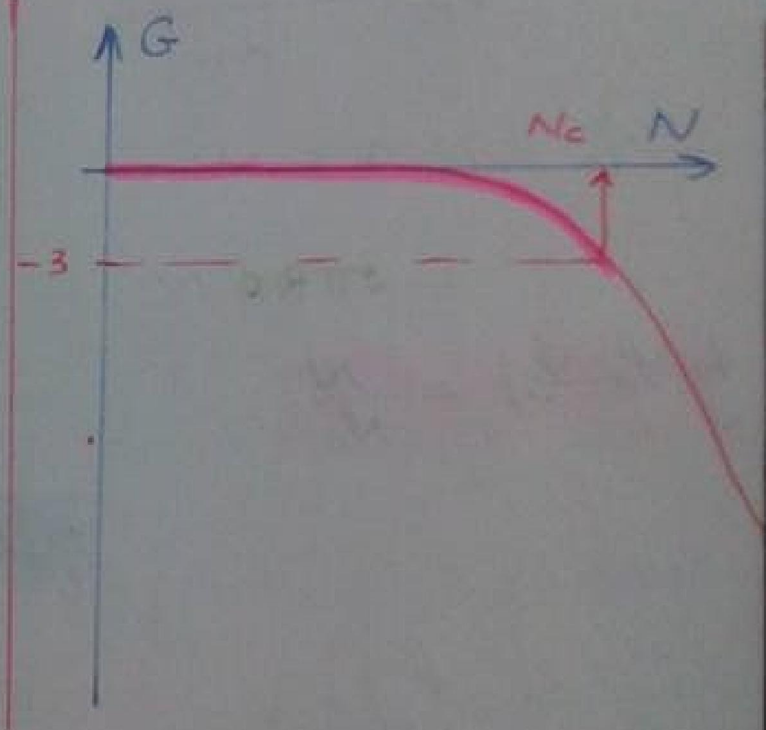
$\varphi_s - \varphi_e = \frac{\pi}{4}$

$U_s(t)$ est en avance de phase % à $U_e(t)$









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Rq : $T = \frac{U_{sm}}{U_{em}}$.

- Si $T > 1$: amplifier le signal
- si $T < 1$, atténué le signal